Candy Boxes by Marilyn Burns

Your Challenge:
You are a box designer for a candy company. Your job is to find all the boxes possible for 24 candies. Boxes must be rectangular and candies are 1-inch cubes.
Candies are packed one layer deep.

Multiple Approaches:
This problem can be used with all ages, kindergarten through fifth grade. It is an engaging problem with constraints built in. It can be used for upper grade children as an introduction to a geometric model for multiplication. Or, for younger children, this problem is an introduction to solving problems using rectangles and addition. Some children use only concrete materials as they look for a solution, others begin with the materials and then move to drawing solutions. Others use their knowledge of multiplication to explore different solutions. Still others challenge themselves to develop a system to make sure they've looked at all the possibilities.

Mathematical Expression and Communication:
Children construct models for each box using blocks or tiles. They build their model out of graph paper. During small and large group discussion, students share their approaches and solutions, often trying to "sell" their solution to others. Older students can write a formal business letter advising the president of the candy company on which box is best (describing each possibility in the process). All children use the language of dimension (2 by 12, 6 by 4) and write their solution symbolically as 2 x 12, 6 x 4.

A Great Problem Engages all Learners
This problem can be simplified by using smaller numbers such as 6 and 12. Younger students can also use 1 inch graph paper to construct their models. The problem can be extended by using a larger number of chocolates and comparing the number of possible boxes. Students can layer arrays instead of working in just two dimensions. Students can also look at the surface area of the boxes of different arrangements to examine how different arrays and layers affect the amount of packaging that would be needed. Many students construct a 3D box out of graph paper, extending their spatial intelligence. Finally, children can be challenged to determine the surface area of the box *without* constructing it.

* And yes, we are aware that the cube above would have 27 pieces of chocolate, well, 26 if you made it hollow in the middle…maybe you could construct it around a core, *then* you'd only need 24 pieces!

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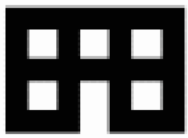
How Many Fingers?

Your Challenge: Determine how many fingers are in the school tonight.

Multiple Approaches: The *How Many Fingers* problem provides students with many opportunities to develop place value and number sense understanding. Students need to gather and identify key information, while activating prior knowledge. Examples of this are how many fingers a person has, how many classes are there in the school, and how many students are in each class. There are multiple entry points from which each student can approach this problem. Some may view this an addition problem, others may immediately choose to use the multiplication algorithm. Still others may still need the assistance of hands-on manipulatives. All of these choices and strategies inform us as educators, assessing where each child is as a mathematical thinker. Students are encouraged to challenge themselves to become more efficient problem solvers.

Mathematical Expression and Communication: The multiple ways children solve problems connects strongly with how they express their learning. Recognizing how they solved a problem, and explaining their thinking is a crucial part of mathematics. Depending on their conceptual understanding of addition, multiplication, and place value children will express their understanding in several ways. You might hear a student say when explaining a mental math strategy, "I know that everyone has 10 fingers and there are twenty children in the class. So, that means there are 200 fingers in each classroom." Younger children may express their problem solving or answer through pictures, tally marks, counters, count by 5 or 10 until they have counted all of the fingers in the building. Older children may begin to use numbers, symbols, and traditional algorithms. Older children also explain their thinking through writing. For example, they might write, "I know there are 180 children in the school and about 15 adults. If I add 180 and 15 I will have 195 people altogether. Then, I take 195 and multiply it by 10 because that is the number of fingers each person has. The makes about 1,950 fingers in the school." The way a child is able to express their thinking gives us a greater window into their minds.

A Great Problem Engages all Learners: As a student progresses as a mathematician, they will choose a variety of ways to approach this problem. For example, as a K-1 student, the problem would be adapted using smaller numbers (such as *How many fingers are in my family?)*, enabling a focus on number sense. Students might be working on one-to one correspondence and grouping numbers in fives. Children may choose to draw pictures in which the finger can be counted. As a 2-3 student, the complexity of the original problem would be increased, including larger numbers. At this level, students may approach the problem as a place value problem, using their knowledge of tens and hundreds to solve the problem more efficiently. Students may choose to do some counting and information gathering in the school to begin their problem solving. For others it becomes a multiplication problem. For 4th and 5th grade students, the complexity of the problem would again be increased and students will often lead discussions of variables (for example, are there any classes on field trips, how many students are out sick). Assessing the way each child solves the problem is key to our future instruction. It allows us as educators to target future lessons to address students at their present level of understanding.



Pentominoes

Your challenge: To find all of the pentomino shapes possible. When you find the total, think about how you will convince the rest of the group you are correct.

Multiple ways to solve the problem:
When we do this problem in a class, some children use trial and error to find the solution. Some children begin with a single line of 5 tiles and move a single tile, then two tiles. Some children draw the possibilities. Some children get tiles and build each possibility – "collecting" them on the table. Some children think in terms of shapes, skinny, wider etc. Our job as teachers is to help students articulate their own method and lead the group in a discussion about mathematical "proof." Can you "prove" that you have not missed any possible shapes.

Mathematic Expression and Communication:
Verbal and visual expression is a key piece to this challenge. A simple number like "18" or "12" is not going to be sufficient to convince others – in fact, both of these numbers are "correct." During the students' discussion, questions such as "do flips count?" open up great opportunities for whole group discussion and debate. If the group decided that flips would not count then the teacher would give a short bit of time for groups to re-configure their solution if necessary. Of course, what a rich way to talk about symmetry when we see that counting flips only increases the number of pentominoes to 18, not 24. A question about "turns" counting could open the area of congruent shapes. We haven't even gotten to systemizing one's exploration yet. The groups that will be most convincing will have a pattern or system that they develop to "test everything."

A Great Problem Engages all Learners:
This challenge never ceases to explode into a fury of pentomino fever. Students become obsessed with exploring with pentominoes. A classic challenge is to make a rectangle using all 12 shapes (flips are fine). Pentacubes is also a neat activity (3-d shapes). The task can be simplified by using tetraminos or triominoes. (Why not dominoes? This is a fun conversation, too). One can extend by challenging students to find hexominoes (when a system of exploration becomes crucial). In addition, some but not all pentominoes can be folded up into an open cube, which ones? How could you cut these pentominoes out of a milk carton?

Grandmas and Acrobats (Problem by Marilyn Burns)

Multiple Strategies:

The Grandmas and Acrobats problem provides opportunities for students to engage in algebraic reasoning. The problem offers a concrete way (tug of war) for students to think about equations, substituting variables of equal value, and holding values constant. Its explicit direction to students to "remember that fact" after telling them that the acrobats (and the grandmas) are of equal strength with each other provides support to students as they develop understandings related to algebra. Some students will substitute variables for the players in the tug of war games. At the fourth and fifth grade level, students often jump to assigning the variables g (for grandma), a (for acrobat), and i (for Ivan). This may be because they already have the understanding of how to use variables, but more likely it is an attempt to use shorthand that facilitates their writing about the problem. If that is the case, then students are getting concrete experience that will prepare them to better understand the use of x and y in their later algebra experiences.

Math Expression and Communication:

In order to express this reasoning, students engage in small group discussion that requires them to offer ideas for how to solve the problem and to provide evidence to support their answer. After students work in small groups to solve the problem, a teacher may call them together as a class so that groups can compare their answers and the methods they used to get them. Another opportunity for math expression comes in the form of writing. Even when students come to the correct answer, it can be a challenge for them to explain their thinking clearly and coherently in writing. Problems such as this provide opportunities for students to practice this important skill.

A Great Problem Engages All Learners

There are multiple ways in which students may engage with this problem. Students with some exposure to formal algebra can use variables and solve for x. Students in the fourth and fifth grades may assign variables to the players (as the attached example shows), but not go so far as to solve for any single variable. Instead, they may substitute one variable or group of variables—in this case, two grandmas and an acrobat—for another variable—in this case, Ivan. Then they can use the information from the three rounds to make valid comparisons. Younger children (Cathy used this problem with third graders) can draw pictures, connect them with lines, and see that certain numbers of players can be "traded equally" for other players. At the K-2 level, developing algebraic thinking is all about developing a sense of numbers and patterns. Very early, children learn to make and extend patterns of shape, color and numbers. They develop a sense of equivalency and inequality. Eventually, children solve equations using number sentences involving addition, subtraction, and unknowns to represent real world situations. Using the idea of rates (if you collect 7 items for a food drive each day, how many will you have on Day 3?) and trades (one dime is worth ten pennies) students can approach algebraic situations.